



## The Royal Society: Mathematical Futures Programme

### Project 2, Work Package 1: An evidence review on the changing nature and importance of mathematics in the 21<sup>st</sup> century

November 2021, Revised January 2022

#### Purpose of this paper

The Royal Society has commissioned Firetail to support the Mathematical Futures Programme, which aims to build a new vision of mathematics education that anticipates and supports the role of mathematics for individuals, economies and society, strengthening diversity and reducing inequity. This paper is the deliverable for the first Work Package of Project 2 of the programme. Project 2 aims to provide evidence and scenarios for the importance and value of mathematics in the future. The purpose of this paper is to provide:

1. Evidence synthesis of studies concerned with changing nature and importance of mathematics in the 21<sup>st</sup> century.
2. Evidence generation for the importance of mathematics to education, society, the economy, and citizenship.

This paper summarises findings from the literature review across three themes: i) mathematics as a discipline; ii) mathematics in work; and iii) mathematics in society. The sources have been proposed by the Firetail team and reviewed by the Royal Society; they include both peer-reviewed articles and grey literature.

This will contribute to Work Package 2 of the project as:

- *A 'current state' analysis as a platform for thinking about the future:* to synthesise evidence on the current role of mathematics in society – to help us identify trends, drivers and axes of uncertainty, and to provide foundations upon which scenarios can build.
- *A stimulus for participants in the scenario planning exercises:* i) to share with participants in interviews and workshops to stimulate thinking on the future importance and role of mathematics, and ii) to support discussions about the current importance of mathematics in order to think about future trends.
- *A summary of any gaps in the evidence base:* to provide theses and frameworks to test in interviews, and explore further gaps in the evidence base.
- *A framework for stakeholder engagement:* to update stakeholders (funders, contact groups) on activity in the MFP programme.

#### Methodology

Sources were purposively sampled based on a combination of:

1. Recommendations from experts in mathematics research and mathematics education.
2. Sources derived from the BEI (British Education Index) research database applying the search terms "mathematics" or "numeracy" and either "adult", "workplace" or "future". The criteria for inclusion were substantive papers published since 2010 that developed a theoretical model and/or presented implications for policy.
3. Following up references from key papers and authors.

This review is not limited to research papers: we have also included policy documents, white papers and grey literature. Please note, this review is not, and does not claim to be, an



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exhaustive on the topics discussed. It is not a systemic review. We have sought to represent the breadth of debate in this field.

### A note on terminology

The use of terminology to describe mathematical skills is varied within mathematics education and particularly in relation to the mathematics necessary for everyday life, sometimes termed 'numeracy' or 'functional mathematics'. Terminology is not consistently applied: for example, 'numeracy' can be used to describe mathematical skills in terms of both the sense of fluency with numbers and the ability to apply that fluency to problems (cf. the English and Welsh curricula, respectively). The internationalisation of mathematics education has increased the popularity of the term 'mathematical literacy' to refer to the cognitive skills and habits needed for applying elementary mathematics. We have used the terminology adopted by the authors when discussing their work. For full discussion on terminology, see O'Donoghue (2003).

### Introduction

UK learned societies and civil society organisations have described the extent of mathematical skills gaps and the urgency of addressing these gaps in the context of a rapidly changing world, and the challenges of preparing citizens to live in a complex and technological society.

- Between 20% and 78% of adults in the UK have inadequate numeracy skills. Estimates for the extent of numeracy skills gaps vary depending on how an adequate level of skill is defined. According to National Numeracy (2019) only 22% of working age adults in the UK are functionally numerate. This is attributed to negative attitudes towards mathematics rather than a lack of innate talent, and that it is culturally acceptable to be negative about mathematics (National Numeracy 2020). In contrast, Maass et al. (2019) suggest that only 20% of adults are *lacking* in numeracy skills.
- Poor numeracy is estimated to cost the UK economy £20.2 billion per year (central estimate; likely range: £6.7-£32.6 billion), or 1.3% of GDP (Pro Bono Economics, 2014). Research by National Numeracy found that politicians and business leaders underestimate the size of this cost to the economy (National Numeracy, 2019). When the true scale of the cost of poor numeracy was revealed, an overwhelming majority support (99% of MPs and 90% of business leaders) a renewed focus on adult literacy.
- Beyond the benefits to the overall economy, there are many benefits to individuals. The British Academy (2015) describes the need to transform quantitative skills across the UK population to help citizens participate in democratic processes, enhance research and innovation in universities and commerce, and to take advantage of the big data revolution.
- Quantitative skills are fundamental to employability: 7 in 10 employees in the UK need some form of quantitative skills to execute their job, but as many as 58% of those in managerial and professional occupations do not have skills above GSCE (British Academy, 2015).
- In terms of benefits to the UK economy, Deloitte (2012) estimated that mathematical science contributed £208 billion to the economy in 2010. This is based on identifying occupations involved in mathematical science research, allocating these occupations across sectors of the UK economy, and approximating supply chain linkages and consumer spending effect to calculate direct, indirect and induced GVA and employment impact. The top 5 sectors where mathematical science made the biggest contribution were banking and finance (£27bn), computer services (£19bn), pharmaceuticals (£16bn), constructions (£13bn) and public administration and defence (£12bn). They estimated 2.8 million individuals in employment directly due to mathematical science occupations, or around 10% of all jobs in the UK.



## Findings: Mathematics as a discipline

**Aim of this section:** Describe how mathematics itself is evolving, its importance in other disciplines and sectors, and how requirements in mathematical competencies of individuals are changing.

### Key takeaways

- The mathematical sciences are becoming more connected both within its subdisciplines and with other disciplines, leading to advances in many fields and more demand for expertise in mathematical sciences.
- Other disciplines and sectors – in STEM, in data science, in social science and humanities – have growing need for mathematical expertise and skills.
- As the skillsets required for modern life are changing, so are the required mathematical skillsets. Many frameworks for understanding the role and purpose of mathematical proficiency have emerged in the literature. These seek to articulate the value of mathematics for individuals.

### *Advances in mathematical sciences*

The US National Academy of Science's (2013) survey of mathematical science makes the case for the vitality of mathematical sciences in its contribution to areas of science, engineering, industry, technology, economic competitiveness and national security through advances such as the fundamental lemma, protein folding and computational biology, primes in arithmetic progression, hierarchical modelling, uncertainty quantification and new frontiers of statistical inference. The survey examines three trends that are shaping mathematical science's growth to 2025:

- Greater connectivity for mathematical science**, both within the discipline itself (e.g. resolutions of the Poincare conjecture) and in research that is applied to other fields of science, engineering, business and medicine (e.g. with theoretical high energy physics and kinetic theory).
- Innovations in modes for scholarly interactions and professional growth**, such as the global growth in mathematical research institutes that have facilitated collaboration that bridges fields and initiate new areas of research.
- Embracing computing / scientific computing as the route for mathematical sciences to be applied to other fields**, for example in creating mathematical and statistical models, validating computational simulations, and exploration of more and better computer-generated data.

Bond (2018) argues that the demand for mathematical science expertise is booming to support fields as varied as finance, AI and machine learning (ML), genomics, autonomous vehicle development, robotics, data science and the digital economy. He argues that mathematics is the single most important driver of innovation in a UK economy that relies on innovation for productivity and rising living standards. To meet the demand for mathematical expertise, this will require the UK to produce 100 more mathematics PhDs per year with stronger emphasis on business and computer coding skills (in addition to c.655 produced in 2015).

### *Interaction with skills needs of other disciplines*

Some research has explored the importance of mathematics and mathematical skills to particular sectors or domains of knowledge. For example:



- **STEM:** the Industrial Strategy Council UK has modelled skills requirements for the UK to 2030. They acknowledge that the knowledge shortages are already most prevalent in STEM subjects. By 2030, acute knowledge shortages are likely to affect a number of people in specific STEM-related occupations – 2.7 million are likely to be affected by under-skilling in at least one STEM subject.
- **Data science:** the Royal Society has convened experts to discuss the need for data analysts capable of deep mathematical and statistical rigour (along with many other non-mathematical competencies e.g. creativity, interdisciplinarity). This has potential to make companies more innovative and productive but requires a skilled workforce. (Royal Society & Royal Statistical Society, 2016).
- **Social science and humanities:** to maintain the UK’s standing as a world class centre for research in social sciences and humanities, universities and employers need to send strong messages about the importance of quantitative skills to combat the imbalance between the growing importance of quantitative skills and declining competence and confidence in quantitative skills (British Academy, 2012)

*Changing requirements in mathematical competencies*

Across the literature, there is increasing emphasis on understanding numeracy and quantitative skills as social practices, and the cognitive rather than procedural abilities required in modern life. Older distinctions between ‘pure’ and ‘applied’ mathematics are eschewed. Bond (2018) prefers ‘impactful mathematics’, as any mathematical method that has practical application generates societal and/or economic value.

Example frameworks for understanding mathematical / numeracy competencies include:

	Source	Scope	Dimensions / categories
Domains for numeracy	Steen (1990)	Purposes of numeracy	Practical, professional, civic, recreational, cultural
	Gal (1997)	Contexts for the use of numeracy	Home, recreation, active parenting, personal finance, informed citizenship, social action, workplace, passing tests, and further education
	Gal et al. (2020)	Domains for numeracy	Financial, health, digital, civic, and workplace numeracy
Nature of numeracy within domains	Tout (2020)	Underpinning facets of numeracy	Cognitive processes, mathematical content, representation, and understanding contexts
	Van Der Wal et al. (2016)	Techno-mathematical literacies used by engineers ( <i>example of specific category 4 typology</i> )	Data literacy, technical software skills, technical communication skills, sense of error, sense of number, technical creativity, technical drawing skills

To expand on one example: Gal et al. (2020) offer an overview of five domains of study for adult numeracy, related to how the ways people cope with the many mathematical, quantitative, and statistical demands of adult life.



1. **Financial numeracy:** critical evaluation of financial information, data, and risk
2. **Health numeracy:** accessing, interpreting, communicating, and acting on numerical, quantitative, and probability-based health information
3. **Digital numeracy:** skills required to understand the impact of facets of today's quantitative, data-rich world such as algorithmic decision-making, the use of big data in making predictions and ubiquitous data collection
4. **Civic numeracy:** monitoring and engaging with social issues by understanding statistical data, and using quantitative knowledge to understand fair distribution and/or fair treatment
5. **Workplace numeracy:** very much a field in its own right, but essentially the many contextually-sensitive numeracy tasks undertaken in workplace

The New Zealand Royal Society has recently published findings from its Advisory Panel on Mathematics and Statistics (2021). Its aims included understanding what mathematics learners need to in order to critically engage with society. Examples of capabilities align with the domains outline by Gal et al. (2020) above, including: understanding risks and probabilities in health, understand uncertainty in future projections, the ability to manage a budget, being a critical consumer of data, and understanding the difference between correlation and causation. It argues that since mathematics is so important to outcomes in life, supporting mathematical education is vital to support equity and justice.

### Findings: Mathematics in the workplace

**Aim of this section:** describe how mathematics skillsets in the workplace are changing, the drivers of this, how skillsets are conceptualised in the literature, what mathematical skills mean in increasingly technological workplaces and what future skills might look like.

#### Key takeaways

- The workplace is a vital context for developing mathematical skills.
- There is substantial body of literature related to the role of mathematics in the workplace, the skills that are needed, exploration of the drivers of changing needs and future requirements. Workplaces require techno-mathematical literacies i.e. mathematical skills that are intertwined with the use of technologies and particular workplace contexts. Their general conclusion is that mathematical skills go beyond procedural efficiency in mathematical operations and are linked to context-specific skills.
- Future mathematical skills in this context relate less to procedural efficiency and more to recognising the applicability of mathematics and translating real-world problems into mathematics for machines to solve.

The Royal Society's own research (2011) has emphasized that there is a steady shift away from manual and low-skill jobs in the UK workforce towards those requiring higher levels of management expertise and problem-solving skills. Many of these are mathematical in nature. It emphasises the importance of the workforce having a more advanced level of skills than required for any particular situation, in order to be confident in using mathematics in unfamiliar situations. Redmer et al. (2019) describe one driver of changes in employment structures changing the relevance of numeracy in working environments as the polarisation of employment. There are more higher- and lower-skilled jobs, and fewer medium-skilled jobs caused by deindustrialisation and automation. The New Zealand Royal Society's recent report (2021) posited three trends in the future of mathematics and statistics and how these are taught: the growth in computational resources (in both capacity and methods), the exponential growth in available data, and the increased presence of artificial intelligence in society. The



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report underlines the importance of giving students age-appropriate tasks to develop computational thinking.

There are two categories of studies on the role of numeracy at work:

- i. large scale international studies (e.g. PISA, PIAAC) which seek to understand the levels of mathematical competencies across populations in OECD countries, and
- ii. ethnographic/qualitative case studies in specific fields.

In examples of the latter, Gravemeijer (2017) identifies research on workplace mathematics used by nurses, carpenters, pool builders, masons, builders and structural engineers. With regards to the former, much discussion revolves around how international datasets' definitions of work and numeracy are evolving to accommodate the changing requirements of modern workplaces and citizenship.

The workplace is a vital context for supporting numeracy. OECD's Survey of adult skills (Jonas, 2018) found that regular engagement with numeracy improves performance, and proficient adults engage with numeracy regularly. The intensity of use of numeracy in everyday life decreases as the time since a person's study increases and employed people engage in mathematics literacy less in the private setting if they do not do so intensively in the workplace. A Department for Business Innovation and Skills study (2016) found that employers have poor awareness of the literacy and numeracy gaps in their workforce and the impact of these on employees' performance.

Duchhardt et al. (2017) have researched the frequency with which 'classic' mathematical concepts are used in the workplace and everyday life across low, normal and advanced users:

- *Little use*: mostly restricted to adding, subtracting, multiplication and some measurement
- *Normal use* (the largest group): all of the above, with fractions, statistics, area and volume calculations
- *Advanced use*: linear functions, algebra, probability calculation, differential and integral calculus and trigonometry

Techno-mathematical literacies (TMLs) is an important concept developed by Hoyles (2010) to consider the role of mathematics in modern ICT-based workplaces. The ubiquity of ICT change in workplaces changes the nature of required mathematical skills, i.e. skills that transcend mathematical knowledge and content. TMLs are combinations of mathematical, ICT and workplace-specific competencies that demand an ability to deal with models and make decisions based on the interpretation of abstract information. Examples of TMLs, collated by van der Wal et al. (2016), include the ability to interpret abstract data, having a sense of number and a sense of error. Mathematical education in some systems lack work- and context-specific problem solving. Estimation and use of software is rarely used, together leading to a gap between school and work mathematics.

FitzSimons (2014) distinguishes between vertical and horizontal discourses as a theoretical distinction in workplace research. Vertical discourse is disciplinary knowledge (theoretical, conceptual and generalisable knowledge, such as formal mathematics). Horizontal discourse on the other hand is contextual, specific, and practical knowledge (such as mathematics used in the workplace). This takes the understanding of mathematics in the workplace beyond simplistic notions of 'mathematics with applications' since contextual knowledge is weakly classified and involves technical skills, workplace knowledge and behavioural skills. These skills take time to develop. It goes some way in explaining why those leaving the education systems are not as workplace ready as employers would like: they have vertical skills but lack horizontal skills.



Gravemeijer et al. (2017) argue that the role of mathematics in our society is growing, but is increasingly and invisibly done by machines. Teaching needs to shift from competencies that compete with computers to competencies that complement computers: less emphasis on solving mathematical problems and more on recognising mathematical applicability, translating reality into mathematics and interpreting outcomes. Their framework describes future competencies in three categories:

1. **Applying and modelling:** translating real world problems into mathematical problems, and translating/presenting results back again by decoding information, structuring problems, making inferences and assumptions, formulating models, presenting results and providing information.
2. **Conceptual understanding:** knowing how mathematical 'black boxes' work to communicate with customers and clients, using the kind of mathematical knowledge and contextual understanding described in TMLs.
3. **Checking:** assessing the mathematical correctness of a computed result, using 'global arithmetic' to simplify calculations to see if results seem reasonable.

Some authors present more specific competencies to describe new skill requirements. Straesser (2015) offers an expanded list of competencies which integrate mathematics and ICT skills which include: ability to create a formula, calculate and estimate, proportional reasoning, calculate and understand percentages correctly, multi-step problem solving, sense of complex modelling and communication of maths to other users. Detecting mathematics in the workplace is difficult since it is challenging to deconstruct mathematical 'black boxes' into parts and recognise the mathematics being used. Beyond 'basic' arithmetic skills, mathematics at work is domain-specific.

Maass et al. (2019) argue that international assessments show that students lack sufficient skills in mathematics and STEM which are critical for personal scientific literacy, international economic competitiveness, and as a foundation for responsible citizenship. They outline three approaches to advancing the role of mathematics within STEM:

1. **21<sup>st</sup> Century Skills** (i.e. abilities and dispositions required for success in a rapidly changing and digital society and workplaces): like STEM, these skills emphasise capabilities such as critical thinking, problem solving and analytical skills, which are enhanced through the use of mathematics as a source of evidence.
2. **Mathematical modelling:** mathematics makes a vital contribution in helping students to understand and make predictions about the world through modelling. Modelling skills make STEM subjects more tangible and relevant to students' lives and their introduction to the world of work, and raises interest in STEM-related careers.
3. **Responsible citizenship:** young people need to be able to understand social and global challenges, such as climate change, food provision, waste reduction, and make informed choices. They need competencies to understand the mathematical formats through which these issues are reported and discussed, e.g. in understanding numerical claims and graphical representations.

### Findings: Mathematics in society / the world

**Aim of this section:** Explore mathematics' applications in society, looking at its contribution in economic terms and its contribution to fields of human endeavour, and how this contribution is predicted to change. It also looks at the contribution of mathematics to human wellbeing in terms of the benefits of mathematical proficiency as well as the harms of lacking mathematical proficiency.



### Key takeaways

- Mathematics has a huge and diverse influence at the cutting edge of human endeavour, which is typically described through examples and case studies; there is no exhaustive review of its many contributions to innovation.
- There is a strong body of evidence related to the benefits of numeracy and quantitative skills, as well as the costs of poor numeracy across many aspects of a citizen's life, from wealth and employment to psychological wellbeing.
- There is a gender dimension to gaps in mathematical skills, and early evidence that the pandemic is intensifying challenges in how mathematical skills are shared.

### Contribution of mathematics to innovation

Many sources discuss the contribution of mathematics to innovation in the 21<sup>st</sup> Century, which are typically illustrated through case studies to show how mathematical science underpins innovations in different industries. For example:

**Smith Review (2017):** examples of contribution of mathematics in healthcare, meteorology, entertainment, finance, transport

Forecasting and simulation for weather systems	Development of special effects in entertainment
Use of big data in healthcare to aid diagnosis, accelerate treatment development and improve patient monitoring	Simulation tools for training e.g. pilots
	AI and ML in used to develop smart infrastructure and autonomous vehicles, and in high frequency trading

**Bond (2018):** examples drawn from trade and finance, healthcare, operational research, national defence, environment, smart infrastructure

Secure transactions/comms through cryptography	Statistics used in genetic analysis to support health and personalised medicine
Smart cities, city planning	National defence uses engineering / tech underpinned by mathematics
Use in search algorithms	Operational analysis to understand worker productivity
Identification of risks / pricing in investment funds	Flood management, meteorology and understanding risks of natural hazards
Underpins digital, medical and environmental innovation	Food security, sustainability
Scheduling airline flights	

The Society for Industrial and Applied Mathematics (2012) survey and case studies of the application of mathematics in industry is as close to a systematic review as we were able to find for this review. The core trend identified is the shift from a product- to a knowledge-based economy where mathematical and computational sciences contribute to new knowledge and new ways of doing business in industries, such as:

- **Business analytics:** use of algorithms and techniques to handle large amounts of structured and unstructured data at low cost. This supports activities across risk management, product design, HR, marketing, and supply chain management.
- **Finance:** use of ever more realistic models in decision make e.g. in algorithmic trading.
- **Systems biology:** genomics, protein folding, population health modelling.
- **Oil discovery and extraction:** simulations and algorithms to find and extract oil based on seismic data, and to minimise risk of accidents.
- **Manufacturing:** designing prototypes (incl. virtual prototyping), optimising designs, planning production and inventory, and managing supply chains.
- **Communications and transportation:** directing internet traffic and allocating bandwidth, planning logistics.
- **Modelling complex systems:** optimising complex systems such as smart grids, traffic networks, water supply systems, buildings and cities medical information networks, as well as non-linear dynamic systems.





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- **Computer systems, software and IT:** cloud computing, computer vision and imaging, natural language processing, information retrieval and machine learning.

The University of Cambridge runs the Mathematics in Industry Reports [portal](#), a repository of (non-peer-reviewed) working papers for early stage research in industrially relevant mathematics. It organises papers into a broad set of industry categories from aerospace to energy and utilities to sport.

#### *Contribution of mathematics to human wellbeing*

There is strong evidence of a link between proficiency in numeracy and use of numeracy in everyday- and working life (Duchhardt et al., 2017). As Tout's (2020) analysis shows, the **benefits of numeracy** are strong, and at least as important as literacy. People with higher literacy and numeracy skills are significantly more likely to be employed, participate in their community, to experience better health, and engage in further training. They also tend to earn more. Noyes' and Adkins' (2017) analysis of British cohorts of mathematics students found that A-level mathematics offers a wage return of 11% (although this benefit is overshadowed by differences in gender and location). Tout argues that the benefits of numeracy to adults go beyond the ability to participate into the workforce - they shape a person's life as a citizen and community member. Participation in the modern world expects and requires the use of technologies, devices, digital media and communication tools at the home and in the community for processes such as banking, shopping, finding information, negotiating the bureaucratic world and enabling citizens to be critical consumers of information. To take a recent example related to coronavirus, Roozenbeek et al. (2020) showed that having higher numeracy skills (and higher trust in scientists) were associated with lower levels of susceptibility to coronavirus-related misinformation.

Conversely, lack of mathematical skills has costs. Parsons and Bynner's study (2005) found that poor numeracy rather than poor literacy is associated with low economic wellbeing at 30; and that low numeracy for women has more negative effects, even with competent literacy. They found that numeracy skills decline when not used in employment, which creates a vicious cycle: poor numeracy leads to limited employment, which leads to declining numeracy, which leads to more limited employment. They recommend that Government policy should therefore target policy on tackling poor standards of numeracy among the most disadvantaged sections of the female population to counter the risk of social exclusion.

Regarding **financial literacy** in particular, the Financial Conduct Authority's *Financial Lives* survey (2021) explores financial numeracy and confidence in a representative sample of the UK public. It found that 18% of adults rate their confidence in working with numbers in everyday life as low; 60% of these adults also lack confidence in managing money, which affects the kinds of financial products they use. They are more likely to hold high-cost credit and less likely to hold savings, investment and insurance products in comparison with their more confident peers. They also found that 34% of adults have low levels of numeracy involving financial concepts – but that this is not always closely related to confidence with numbers, as 44% of adults are over-confident in their financial numeracy.

There is a dedicated field looking at the psychological costs of low mathematical ability, often referred to as '**mathematics anxiety**' and its relationship to performance in mathematics. The British Academy commissioned a systemic review of mathematics anxiety to understand this phenomenon as a psychological, cultural and sociological phenomenon (British Academy, 2016); the findings have not yet been published. As an example of recent research on this topic, Carey et al. (2019) suggest that mathematics is perceived as being harder than other subjects among UK school children. It appears that the link between mathematical anxiety and



performance becomes more entrenched during late childhood and early adolescence. In their review of research on the topic to date, Dowker et al. (2016) found that mathematics anxiety relates to other forms of anxiety (test anxiety, general anxiety), but is a phenomenon in its own right. Mathematics anxiety also correlates with both lower performance in mathematics and negative attitudes towards mathematics. Where causation lies is an open question.

There is extensive literature on the **gender gap** in STEM education and professions, a subset of which is focused on mathematics and numeracy. Borgonovi's (2021) analysis of international datasets indicates that boys' advantage in numeracy grows into adulthood. This is unlike the advantage that girls have in literacy, where the gap closes by early adulthood. They suggest this is initially attributable to gender norms and psychological traits, and then compounded by educational choices.

In recent research, Bakker et al. (2021) surveyed mathematical education experts to discover future themes in the focus of mathematical education. They found that the pandemic has served as a magnifying glass on issues that need to be better understood, such as the importance of empowering students to combat anti-expertise and post-fact tendencies, concerns about the digital divide when reliance on online teaching is growing, and supporting teachers to develop professionally in a time of immense pressure.

### Commentary on alignment to Royal Society citizen typology

The literature review generally supports the Royal Society classification of users of mathematics insofar as there is alignment between citizen categories and the subjects of the articles discussed here. The table below demonstrates the alignment of different sources to different citizen categories:

RS Category of citizens	Sources
1. Basic functional numeracy for personal decision making	Tout (2020), Gal (2020), Gravemeijer et al. (2017)
2. Mathematical literacy to understand and critique numerical claims	Straesser (2015), Tout (2020), Gravemeijer et al. (2017), Gal (2020), Maass et al. (2019)
3. Traditionally non-quantitative jobs/professions, but requiring mathematical expertise: e.g., lawyers, journalists, civil servants, politicians, health practitioners, administrators, teaching practitioners (not mathematics), technical and vocational professions – including technicians.	Hoyles et al. (2010), Redmer et al. (2019), National Numeracy (2019), Mason et al. (2017), FitzSimon (2014), Gravemeijer et al. (2017), Straesser (2015)
4. Jobs/professions where mathematical competences are a core component, e.g. analysts, engineers, financial professionals, scientists, social scientists.	Bond (2018), Hoyles et al. (2010), British Academy (2012), Royal Statistical Society (2017), Mason et al. (2017), FitzSimon (2014), Maass et al. (2019), Deloitte (2013)
5. Mathematical sciences, e.g. mathematicians, mathematics teachers.	Bond (2018), National Academy of Sciences (2012)

— Most papers discussed here do not attempt to characterise the same breadth of types of citizen. In comparison to the typology, most papers examined here discuss one or two



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- categories, or groups of categories. The various international studies – PIAAC, PISA – categorise whole populations, but they do so on the basis of proficiency rather than need.
- In category (1), many papers expand on domains for personal decision-making. For example, Gal et al. (2020) discuss fields such as financial, health and digital numeracy. They also explore the capabilities that make up functional numeracy.
  - Articles tend to focus on either groups (1) and (2) (categories where mathematical needs are not specifically related to work but more oriented to everyday life), or (3), (4) and (5) (categories where the need is more oriented towards work) – this reflects the extensive literature devoted to workplace mathematics that we have explored in this review.
  - Articles addressing groups (1) and (2) do not strongly differentiate these needs. They are either treated as related, or where (2) is a subset of (1). The categories do acknowledge the difference in the purpose of these needs i.e. mathematics required to function in everyday life and the mathematics to participate as citizens in modern society / critical consumers of data claims.
  - Category (3) could be expansive, depending on how mathematical expertise is defined. It is not clear whether this category includes professions where mathematics is important but basic or repetitive (e.g. for a nurse or a wholesaler). The description of the categories implies that this is a group of professions where mathematical expertise is becoming more important. Whether this is the case could be clarified.
  - Category (5) could be split into two sub-categories to distinguish people who are creating new mathematical knowledge from those who are teaching existing knowledge.

### Emerging themes and questions raised by the literature review

The topics in the span of this literature review are wide-ranging. As such, it is perhaps unsurprising that the majority of researchers cited emphasise that the topic they examine is under-researched. Nonetheless, in the context of this project, there are particular questions raised that need to be explored in the next phase of work:

- **Impact of skills gaps:** there is extensive discussion about the current costs of skills gaps, and the changing mathematical capability requirements. There is however little discussion or speculation of potential future states if these requirements are met or not: i.e. descriptions of what the world might look like if ambitions for mathematical capabilities are realised across the types of citizens. What will happen if the needs of citizens are not met, and what is the prize if they are?
- **Perception of benefits of mathematics:** there is research on the perceptions of mathematics, particularly where negative perceptions have a role in shaping ability through topics like ‘maths anxiety’. There is little research on the awareness and perception of the benefits of acquiring mathematical skills. Is there awareness in society at large on the importance and benefits of mathematical skills?
- **Driver combinations and permutations:** the contribution of mathematics to innovation is anecdotal, or is discussed through proposed drivers that are making mathematics’ influence stronger (e.g. digitisation, intensification of the knowledge-based economy). A complete or more quantitative picture of the contribution would be very ambitious, but an understanding of such a programme of work would be helpful. What drivers can be identified, and how might they combine and evolve?
- **Role of employers:** this review underlines the importance of the workplace and everyday life in supporting mathematical skills, beyond the classroom. There is little exploration of the perspectives of employers in enhancing the mathematical skills of their workforces. What insights can they offer on the contribution of mathematics to industry?



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